Self consistency in hadron physics

M.F.M. Lutz[1,*)], C.L. Korpa[2] and A. Semke[1]

[1] *Gesellschaft für Schwerionenforschung (GSI), Planck Str. 1, 64291 Darmstadt, Germany*
[2] *Department of Theoretical Physics, University of Pecs, Ifjusag u. 6, 7624 Pecs, Hungary*

In this talk we discuss at hand of two examples the crucial role played by self consistency in hadron physics. The first example concerns the quark-mass dependence of the baryon octet and decuplet masses. It is shown that within a self consistent one-loop approach based on the chiral Lagrangian the 'mysterious' quark-mass dependence of the Ξ mass predicted by the MILC collaboration may be recovered in terms of a discontinuous chiral extrapolation. This is a consequence of self consistency imposed on the partial summation, i.e. the masses used in the loop functions are identical to those obtained from the baryon self energies. In the second example we discuss recent studies on the properties of D mesons in cold nuclear matter as they are predicted by coupled-channel dynamics. Here a self consistent many-body approach reveals the close interlink of the properties of D meson and open-charm baryon resonances in nuclear matter. The relevance of exotic baryon resonances for the spectral distortion of the $D_s^\pm$ in nuclear matter is pointed out.

§1. Introduction

In this talk we discuss two seemingly unrelated topics. First the quark-mass dependence of baryon masses and second the properties of D mesons in cold nuclear matter. The two topics exemplify the crucial importance of self consistency in hadron physics. As it will become clear during the talk both topic have exotic aspects justifying its presentation in a workshop about 'exotics'.

The present-day interpretation of QCD lattice simulations requires a profound understanding of the dependence of observable quantities on the light quark masses. A powerful tool to derive such dependencies is the chiral Lagrangian, an effective field theory based on symmetry properties of QCD. The application of strict chiral perturbation theory to the SU(3) flavor sector of QCD is plagued by poor convergence properties for processes involving baryons.[1)–3)] Thus it is important to establish partial summation schemes that enjoy improved convergence properties and that are better suited for chiral extrapolations of lattice simulations. We review the results of[4),5)] where recent lattice QCD simulation of the MILC collaboration,[6),7)] that use dynamical u-,d- and s-quarks in the staggered approximation are interpreted. Adjusting the values of the Q^2 counter terms to the physical masses we suggest the possibility of a discontinuous dependence of the baryon masses on the pion mass. This is a consequence of self consistency imposed on the partial summation approach, i.e. the masses used in the loop function are identical to those obtained from the baryon self energy. The latter is a crucial requirement since the loop

*) e-mail address: m.lutz@gsi.de

functions depend sensitively on the precise values of the baryon masses. Our results may explain the mysterious quark-mass dependence for the Ξ mass observed by the MILC collaboration.[6),7)]

Except for the recent works[8)–10)] a mean field ansatz was assumed to predict the properties of D mesons in nuclear matter.[8),11)–13)] This development resembles to a large extent the first attempts to predict the properties of kaons and antikaons in nuclear matter (see e.g.[14)–16)]), which also assumed a mean-field type behavior for the mass shifts in nuclear matter at first. However, by now it is well established that for the antikaon such an ansatz is not valid due to complicated many-body dynamics induced by the presence of resonance-hole states.[17),18)] A self consistent approach is required that treats the spectral distortions of the hyperon resonances and the antikaon in a consistent manner. As illustrated in[9)] it is crucial to study also the properties of D mesons in a more microscopic manner. The key ingredient of a realistic description are the D-meson nucleon scattering amplitudes, that determine the D meson self energy at least for dilute nuclear matter.[16),19)] In contrast to the D_--nucleon, the D_+-nucleon system may couple to open-charm baryon resonances. This poses a particular challenge since the spectrum of the open-charm baryon resonances is studied experimentally and theoretically poorly so far. As suggested first in[20)] the well established $\Lambda_c(2594)$ s-wave resonance resonance may be dynamically generated by coupled-channel interactions.[8)] A detailed study[21)] that took into account charm-exchange reactions systematically revealed that it couples strongly to the D_--nucleon channel. Thus the $\Lambda_c(2594)$ should play an important role in the description of the nuclear D_- dynamics. Besides the narrow $\Lambda_c(2594)$, which couples dominantly to the $DN, D_s\Lambda$ channels, the work[21)] recovers a broad s-wave state, so far unobserved, that is interpreted as a chiral excitation of the open-charm sextet ground states.[20)] The latter couples strongly to the $\pi\Sigma_c$ channels but very weakly to the DN channel. It is the analogue of the $\Lambda(1405)$, which is a chiral excitation of the baryon octet ground states.[22)–25)] A further striking prediction of the work[21)] is a narrow isospin one resonance of mass 2.62 GeV which couples dominantly to the $DN, D_s\Sigma$ channels. This state is so far unobserved, but, so it existed, would affect the properties of D_+ mesons in nuclear matter significantly.[9)]

§2. Chiral extrapolation of baryon masses

We collect the terms of the chiral Lagrangian that determine the leading orders of baryon octet and decuplet self energies.[26)] Up to chiral order Q^2 the baryon propagators follow from

$$\begin{aligned}
\mathcal{L} &= \mathrm{tr}\left(\bar{B}\left[i\,\partial\!\!\!/ - \overset{\circ}{M}_{[8]}\,\right]B\right) \\
&- \mathrm{tr}\left(\bar{\Delta}_\mu\cdot\left(\left[i\,\partial\!\!\!/ - \overset{\circ}{M}_{[10]}\,\right]g^{\mu\nu} - i\left(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu\right) + \gamma^\mu\left[i\,\partial\!\!\!/ + \overset{\circ}{M}_{[10]}\,\right]\gamma^\nu\right)\Delta_\nu\right) \\
&- 2\,d_0\,\mathrm{tr}\Big(\bar{\Delta}_\mu\cdot\Delta^\mu\Big)\,\mathrm{tr}\Big(\chi_0\Big) - 2\,d_D\,\mathrm{tr}\Big((\bar{\Delta}_\mu\cdot\Delta^\mu)\,\chi_0\Big) \\
&+ 2\,b_0\,\mathrm{tr}\Big(\bar{B}\,B\Big)\,\mathrm{tr}\Big(\chi_0\Big) + 2\,b_F\,\mathrm{tr}\Big(\bar{B}\,[\chi_0,B]\Big) + 2\,b_D\,\mathrm{tr}\Big(\bar{B}\,\{\chi_0,B\}\Big)\,,
\end{aligned}$$

$$\chi_0 = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2\,m_K^2 - m_\pi^2 \end{pmatrix} , \tag{2·1}$$

where we use the notations of[4),5)] for the baryon octet and decuplet fields B and Δ respectively. The evaluation of the baryon self energies to order Q^3 probes the meson-baryon vertices

$$\begin{aligned}\mathcal{L} &= \frac{F}{2f}\,\mathrm{tr}\left(\bar{B}\,\gamma_5\gamma^\mu\,[\partial_\mu\Phi,\, B]\right) + \frac{D}{2f}\,\mathrm{Tr}\left(\bar{B}\,\gamma_5\gamma^\mu\{\partial_\mu\Phi, B\}\right)\\ &- \frac{C}{2f}\,\mathrm{tr}\left(\bar{\Delta}_\mu\cdot(\partial_\nu\Phi)\left[g^{\mu\nu} - \frac{1}{2}\,Z\,\gamma^\mu\,\gamma^\nu\right]B + \mathrm{h.c.}\right)\\ &- \frac{H}{2f}\mathrm{tr}\left(\left[\bar{\Delta}^\mu\cdot\gamma_5\,\gamma_\nu\,\Delta_\mu\right](\partial^\nu\Phi)\right),\end{aligned} \tag{2·2}$$

where we apply the notations of.[27)] We use $f = 92.4$ MeV in this work. The values of the coupling constants F, D, C and H may be correlated by a large-N_c operator analysis.[28)–30)] At leading order the coupling constants can be expressed in terms of F and D only. We employ the values for F and D as suggested in.[27),31)] All together we use

$$F = 0.45\,, \qquad D = 0.80\,, \qquad H = 9\,F - 3\,D\,, \qquad C = 2\,D\,. \tag{2·3}$$

We take the parameter $Z = 0.72$ from a detailed coupled-channel study of meson-baryon scattering that was based on the chiral SU(3) Lagrangian.[27)] The latter parameter is an observable quantity within the chiral SU(3) approach: it contributes at order Q^2 to the meson-baryon scattering amplitudes and cannot be absorbed into the available Q^2 counter terms.[27)]

It is straight forward to evaluate the one-loop contributions to the baryon self energy as implied by (2·2). Explicit and complete expressions are provided for the first time in.[4),5)] Since we do not advocate a strict chiral expansion, rather a partial summation, the renormalization needs to be discussed briefly. As pointed out in,[4)] within dimensional regularization there persists an ambiguity on how to implement the chiral counting rules. This leads to the presence of an infrared renormalization scale that can be used to optimize the speed of convergence. Performing a strict chiral expansion the physical parameters are independent on the infrared scale, as they should be. However, the size of the Q^2 counter terms depend linearly on this scale. In turn the apparent convergence properties reflect the choice of that scale. Only for reasonable values of the infrared scale, μ_{IR}, the counter terms have a natural size.

Once a partial summation, as implied by a self consistent evaluation of the loop function, is performed an infinite number of counter terms would be needed to arrive at results that are manifestly independent on the renormalization scales. Dialing a value of the infrared scale amounts to a particular choice thereof. Any residual dependence of the baryon masses on the infrared scale may be used to estimate the uncertainty of the given truncation.

	$\mu_{IR} = 350$ MeV	$\mu_{IR} = 450$ MeV	$\mu_{IR} = 550$ MeV
b_0 [GeV^{-1}]	-0.89	-0.63	-0.38
b_D [GeV^{-1}]	$+0.29$	$+0.19$	$+0.10$
b_F [GeV^{-1}]	-0.34	-0.25	-0.15
d_0 [GeV^{-1}]	-0.22	-0.15	-0.08
d_D [GeV^{-1}]	-0.35	-0.30	-0.24
M_N [MeV]	$750 + 310 - 121 = 939$	$813 + 232 - 105 = 939$	$875 + 153 - 89 = 939$
M_Λ [MeV]	$750 + 536 - 150 = 1136$	$813 + 398 - 79 = 1131$	$875 + 260 - 9 = 1126$
M_Σ [MeV]	$750 + 882 - 425 = 1207$	$813 + 630 - 239 = 1203$	$875 + 379 - 54 = 1200$
M_Ξ [MeV]	$750 + 934 - 361 = 1323$	$813 + 680 - 171 = 1321$	$875 + 427 + 19 = 1320$
M_Δ [MeV]	$1082 + 241 - 91 = 1232$	$1108 + 164 - 39 = 1232$	$1133 + 87 + 12 = 1232$
M_Σ [MeV]	$1082 + 347 - 49 = 1380$	$1108 + 253 + 17 = 1378$	$1133 + 160 + 83 = 1376$
M_Ξ [MeV]	$1082 + 453 - 4 = 1530$	$1108 + 343 + 78 = 1528$	$1133 + 233 + 160 = 1526$
M_Ω [MeV]	$1082 + 558 + 34 = 1674$	$1108 + 432 + 134 = 1674$	$1133 + 305 + 235 = 1674$
$\sigma_{\pi N}$ [MeV]	52.4	53.9	55.7
$\sigma_{K^- p}$ [MeV]	384.0	380.1	386.3
$\sigma_{K^- n}$ [MeV]	359.4	354.8	361.5

Table I. The parameters are fitted to reproduce the baryon masses at physical pion masses as well as the SU(3) limit values $M_{[8]} \simeq 1575$ MeV and $M_{[10]} \simeq 1710$ MeV at $m_\pi \simeq 690$ MeV. We use $\mu_{UV} = 800$ MeV. The masses are decomposed into their chiral moments.

For given values of the infrared and ultraviolet renormalization scales the parameters b_D, b_F and d_D are fitted to the mass differences of the octet states and decuplet states. The absolute mass scale of the octet and decuplet states can be reproduced by appropriate values of the bare baryon masses. This procedure leaves undetermined the two parameters b_0 and d_0. Good representations of the physical baryon masses can be obtained for a wide range of the latter. The parameter b_0 may be used to dial a given pion-nucleon sigma term at physical pion masses. Similarly the unknown parameter d_0 may be determined to reproduce a given pion-delta sigma term. The latter two parameters are adjusted as to reproduce the baryon octet and decuplet masses in the SU(3) limit at $m_\pi \simeq 690$ MeV. The MILC simulations suggest the values $M_{[8]} \simeq 1575$ MeV and $M_{[10]} \simeq 1710$ MeV. This procedure is biased to the extent that it assumes that the self consistent one-loop results will be applicable at such high quarks masses. On the other hand as long as there are no continuum limit results of the MILC collaboration available this is an economical way to minimize the influence of lattice size effects. The latter are expected to be smaller at large quark masses. The procedure may be justified in retrospect if it turns out that the extrapolation recovers the behavior predicted by the lattice simulation.

The parameters used are collected in Tab. I together with the implied masses of

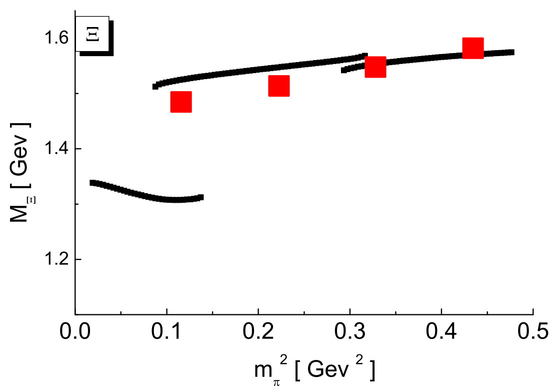

Fig. 1. The pion mass dependence of baryon octet and decuplet masses predicted by the chiral loop expansion taking the parameters of Tab. I. The lines represent the masses for the infrared scale put at $\mu_{IR} = 450$ MeV. The solid squares are the simulation points of the MILC collaboration.

the baryon octet and decuplet states. A fair representation of the physical baryon masses is obtained. As discussed in detail in[4] the parameters $b_{0,D,F}$ and $d_{0,D}$ show a strong dependence on the infrared scale μ_{IR}. In contrast, the physical baryon masses suffer from a weak dependence only. For natural values of the infrared scale the chiral expansion appears well converging as indicated by the decomposition of the baryon masses into their moments. Taking the residual scale dependence of the pion-nucleon sigma term as a naive error estimate we obtain $\sigma_{\pi N} = 54 \pm 2$ MeV.

We turn to the pion-mass dependence of the baryon octet and decuplet masses. It should be emphasized that the baryon masses are a solution of a set of coupled and non-linear equations in the present scheme. This is a consequence of self consistency imposed on the partial summation approach. As a consequence of the non-linearity for a given parameter set there is neither a guarantee for a unique solution to exist, nor that solutions found are continuous in the quark masses. Indeed, as illustrated by Fig. 1 the pion-mass dependence predicted by the chiral loop expansion is quite non-trivial exhibiting various discontinuities. It is striking to see that we reproduce the 'mysterious' pion-mass dependence of the Ξ mass, i.e. the quite flat behavior which does not seem to smoothly approach the physical mass. Given the present uncertainties from finite lattice spacing, the staggered approximation and the theoretical uncertainties implied by higher order contributions, we would argue that we arrive at a fair representation of the lattice simulation points for all baryons (see[5]) with some reservation concerning the nucleon. Incorporating the many Q^4 counter terms offered by the chiral Lagrangian it is reasonable to expect that the latter will further improve the picture. However, as long as there is no detailed analysis available that performs the continuum limit there is not much point considering the Q^4 counter terms.

§3. D mesons in nuclear matter

After a demonstration of the crucial importance of self consistency for the chiral extrapolation of the baryon octet and decuplet masses we turn to the propagation of D mesons in a cold nuclear environment. We discuss the properties of the $D^{\pm}$ and $D_s^{\pm}$ mesons as derived from a self consistent many body approach based on coupled-channel dynamics.[9] The dominant interaction was modelled by the exchange of light vector mesons in the t-channel.[21] All relevant coupling constants were obtained from chiral and large-N_c properties of QCD. Less relevant three-point vertices related to the t-channel forces induced by the exchange of charmed vector mesons were estimated by a flavor SU(4) ansatz.[21] The resulting s-wave $D_- N \to D_- N$ scattering amplitudes are dominated by the dynamically generated $\Lambda_c(2594)$ and $\Sigma_c(2620)$ resonances. The s-wave $D_s^{\pm} N \to D_s^{\pm} N$ scattering amplitudes are characterized by so far unobserved exotic resonances at 2.89 GeV and 2.78 GeV. Only the $D_- N \to D_- N$ scattering process is not influenced by the presence of a resonance. In this case the amplitude is characterized to a large extent by the scattering length

$$a^{(I=0)}_{D^- N} \simeq -0.16\,\mathrm{fm}\,, \qquad a^{(I=1)}_{D^- N} \simeq -0.26\,\mathrm{fm}\,. \tag{3·1}$$

The many-body computations are based on the self consistent and covariant formalism established in.[17),18)] It is important to perform such computations in a self consistent manner since the feedback of an altered meson spectral function on the resonance structure is typically an important effect.[17),18),32)] Numerical results for the spectral distributions of the D mesons as derived in[9] are shown in Fig. 2. Given the scattering lengths (3·1) the mass shift of a D_- meson in nuclear matter

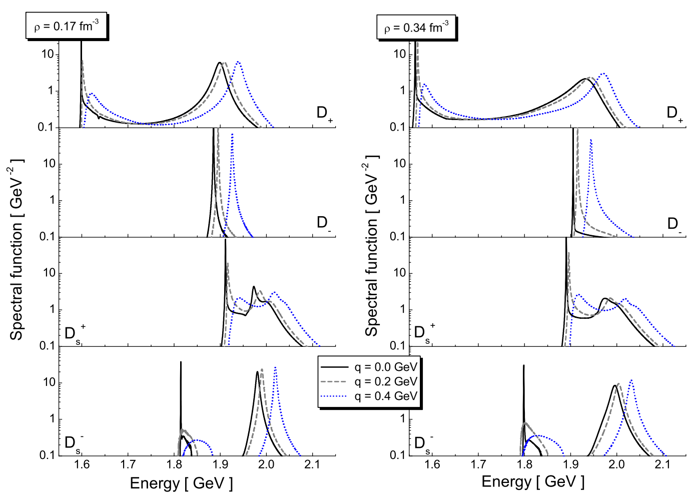

Fig. 2. Spectral distributions of the $D^{\pm}$ and $D_s^{\pm}$ mesons. Results are shown for the meson momenta 0, 200 and 400 MeV and nuclear densities 0.17 fm^{-3} and 0.34 fm^{-3}. The self consistent many-body approach of[18] was applied.

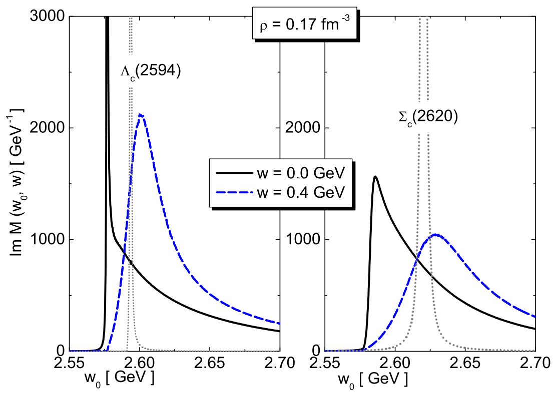

Fig. 3. Spectral distributions of the $D^{\pm}$ and $D_s^{\pm}$ mesons. Imaginary part of the isospin zero (l.h.p.) and isospin one (r.h.p) D_+-nucleon scattering amplitude at saturation density as compared to the free-space case (dotted lines). The amplitudes are shown for two values of the resonance three-momentum $w = 0$ GeV and $w = 0.4$ GeV.

is fully determined by the low-density theorem.[16),19)] The low-density mass shift of 17 MeV is quite close to the self consistent result shown in Fig. 2. Self-consistency leads to a mass shift of 18 MeV at saturation density and 38 MeV at twice saturation density. The spectral function of the D_+ meson has a two-mode structure, which is a consequence of important resonance-hole contributions. At saturation density the main mode is pushed up by about 32 MeV as compared to the free-space meson. Our results for the D_+ meson differ from the previous study[8)] significantly. This is a consequence of the quite different interaction used in the two computations. The work[8)] overestimates the charm-exchange channels. In[21)] those channels are suppressed by a kinematical factor $m_\rho^2/m_D^2 \sim 0.2$. In particular the work[8)] did not predict the isospin one resonance $\Sigma_c(2620)$. The latter dominates the resonance-hole component in the spectral distribution.

We turn to the mesons with non-zero strangeness. The study of[21)] predicts the existence of a coupled-channel molecule in the ($D_s^- N$, $\bar{D}\Lambda$, $\bar{D}\Sigma$) system. The latter carries exotic quantum numbers that can not be arranged by three quarks only. Exotic s-wave states with C=-1 were discussed first by Gignoux, Silvestre-Brac and Richard[33)] and later by Lipkin.[34)] The binding of about 190 MeV for such a the state as predicted in[21)] awaits experimental confirmation. If confirmed the s-wave $D_s^- N \to D_s^- N$ scattering amplitude must show a prominent pole structure at subthreshold energies. The latter leads to a well separated two-mode structure of the D_s^- spectral distribution. The main mode is pushed up by less than 10 MeV at nuclear saturation density.

The in-medium spectral distribution of the D_s^+ is most striking. The additional mode expected from the possible existence of the exotic state at 2.89 GeV merges with the main D_s^+ mode into one broad structure, in particular at intermediate meson momenta 400 MeV. The results are quite analogous to the spectral distribution of the K_- where the $\Lambda(1405)$ nucleon-hole state gives rise to a broad distribution.[17),18)]

Clearly this result is an immediate consequence of the small binding energy of the exotic state.

In Fig. 3 the spectral functions of the $\Lambda_c(2594)$ and $\Sigma_c(2620)$ resonances are shown at saturation density as compared to their free-space distributions. We observe small attractive shifts in their mass distributions and significant broadening, at least once the resonances move relative to the matter bulk. The shifts are the result of a subtle balance of the repulsive Pauli blocking effect and attraction implied by the coupling of the D_+ meson to resonance-hole modes.

§4. Conclusions

The pion-mass dependence of the baryon octet and decuplet masses was discussed. A novel approach was reviewed in which the latter are a solution of a set of coupled and non-linear algebraic equations. This is a direct consequence of self consistency imposed on the partial summation, i.e. the masses used in the loop functions are identical to those obtained from the baryon self energies. As a striking consequence a discontinuous dependence of the baryon masses on the pion mass arises. Typically the baryon masses jump at pion masses as low as 300 MeV. Most spectacular is the behavior of the Ξ mass. At small pion masses it decreases with increasing pion masses. At a critical pion mass of about 300-400 MeV it jumps up to a value amazingly close to the prediction of the MILC collaboration.

In the second part of this talk we discussed recent results on the properties of $D_\pm$ and $D_s^\pm$ mesons in cold nuclear matter based on self consistent coupled-channel dynamics. It was pointed out that the $D_+, D_s^\pm$ spectral distributions are strongly distorted in a nuclear medium due to the presence of resonances that couple strongly to the $D_+, D_s^\pm$ nucleon channel. Since even the existence of most resonances that are predicted by coupled-channel dynamics is not yet confirmed experimentally such studies suffer from large uncertainties. This asks for a dedicated experimental programm to unravel the spectrum of baryons with charm content.

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